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Characterization of Magneto-Dielectric Materials by a Rectangular Waveguide Using the 2D-FDTD Method

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***Abstract*- In this paper, a new measurement method is proposed to estimate simultaneously the complex permittivity and the complex permeability for a magneto-dielectric sample, which may contain several layers, using a X-band rectangular waveguide WR90. The Sij-parameters at the reference planes in the rectangular waveguide loaded by a material sample are measured as a function of frequency using the E8634A Vector Network Analyzer. Also, by applying the two-dimensional finite difference in time domain (2D-FDTD), the expressions for these parameters as a function of complex permittivity and complex permeability are calculated even if the sample do not cover all the section of the waveguide. The Nelder-Mead algorithm is then used to estimate the complex permittivity and complex permeability by matching the measured and calculated S_{ij}-parameters. This method has been validated by estimating, at the X-band, the complex permittivity and complex permeability of two materials such as FR4 Epoxy and Titanium Carbide powder (TiC).**

***Index Terms*- Electromagnetic characterization, microwave, rectangular waveguide, Nelder-Mead algorithm, FDTD-2D method.**

I. INTRODUCTION

The last decades have seen, on the one hand, the emergence of new technologies microwave integrated circuits where the passive components (inductors, capacitors, resistors) are widely used but very bulky [1,2]. The need for integration increases in these telecommunication systems is reflected in the direct implantation above integrated circuits. The use of strongly soft magnetic materials permeable in inductors and energy converters, as well as in materials piezoelectric or/and ferroelectric with high dielectric constant in capacitors and gates of transistors, will be a good solution and arouses new microwave applications [3].

On the other hand, the increase in electromagnetic pollution caused by the drastic development of electronic and telecommunications systems. The materials X-band electromagnetic, magnetic or dielectric wave absorbers, have much attracted the interest of researchers [4-6].

In this paper, we propose to apply the 2D-FDTD method to simultaneously estimate the complex permittivity ε^* and the complex permeability μ^* of a magneto-dielectric material placed in a standard X-band rectangular waveguide WR90. This numerical method avoids to use the theory line transmission which is very complicated if the sample is multilayer or not cover the full section of the waveguide.

The parameters S_{ij} at the reference planes are measured as a function of the frequency. The expressions of these parameters as a function of the complex permittivity and the permeability complex are calculated, by applying the 2D-FDTD method. Then the Nelder-Mead algorithm is used in order to estimate the values of (ε^*,μ^*) by matching the S_{ij} parameters measured and calculated. This method was validated by estimating, at the X band, of the complex permittivity and permeability of a non-magnetic dielectric material as FR4 Epoxy [7] and a magnetic material such as Titanium Carbide powder (TiC) [6].

II. THEORY

In a rectangular waveguide of width (a), a sample of material magnetic with complex permittivity eps and complex permeability mu, of thickness (L), is placed at a distance (L_0) from the two reference planes P_1 and P_2 in Fig. 1.

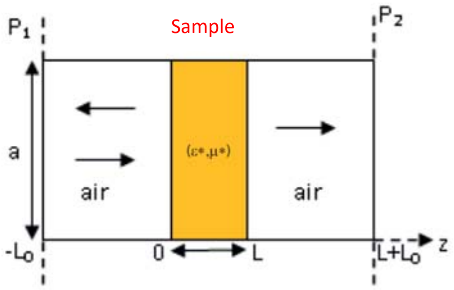

Fig.1. Waveguide charged by a dielectric and/or magnetic single-layer.

A. Direct Problem

The waveguide is only excited by a dominant TE_{10} mode. The electric field inside the rectangular waveguide is obtained using the 2D-FDTD method. The 2D-FDTD formulation is based on a direct discretization of the equations of Maxwell given by [8]:

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu^*} \vec{\nabla} \times \vec{E} \tag{1}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon^*} \vec{\nabla} \times \vec{H} \tag{2}$$

Where $\varepsilon^*=\varepsilon'-j\varepsilon''$ and $\mu^*=\mu'-j\mu''$ present the complex permittivity and permeability complex. In Cartesian coordinates and following the notation of Yee, we obtain the two-dimensional FDTD formulation, for the components E_y, H_x and H_z in the equations (1) and (2):

$$H_x^{n+\frac{1}{2}}(i,k) = H_x^{n-\frac{1}{2}}(i,k) + \Delta t \left[\frac{E_y^n(i,k+1) - E_y^n(i,k)}{\mu^*(i,k)\Delta z}\right] \tag{3}$$

$$H_z^{n+\frac{1}{2}}(i,k) = H_z^{n-\frac{1}{2}}(i,k) - \Delta t \left[\frac{E_y^n(i+1,k) - E_y^n(i,k)}{\mu^*(i,k)\Delta x}\right] \tag{4}$$

$$E_y^{n+1}(i,k) = E_y^n(i,k) + \Delta t \left[\frac{H_x^{n+\frac{1}{2}}(i,k) - H_x^{n+\frac{1}{2}}(i,k-1)}{\varepsilon^*(i,k)\Delta z} - \frac{H_z^{n+\frac{1}{2}}(i,k) - H_z^{n+\frac{1}{2}}(i-1,k)}{\varepsilon^*(i,k)\Delta x}\right] \tag{5}$$

Where Δx and Δz are the space steps in the two directions x and z, and Δt is the temporal. In order to ensure the convergence of the program, it is necessary that the meshes of the 2D-FDTD network are sufficiently small compared to the wavelength in the guide.

$$\frac{\lambda_{gmin}}{100} < \mathrm{Max}(\Delta x, \Delta z) < \frac{\lambda_{gmin}}{10} \tag{6}$$

With: $$\lambda_{gmin} = \frac{1}{\sqrt{\varepsilon_r \mu_r \frac{f_{max}^2}{c^2} - \frac{1}{(2a)^2}}} \tag{7}$$

To ensure the numerical stability of the 2D-FDTD algorithm, we choose the Δt, Δx and Δz increments such that [9]:

$$\Delta t = \frac{0.9}{c}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}\right)^{-\frac{1}{2}} \tag{8}$$

Where c is the speed of light in a vacuum.

To obtain a set of Maxwell's equations that can be solved numerically, the boundary conditions must be defined. In particular, the components tangential elements of the electric field and the magnetic field are continuous through a dielectric and/or magnetic interface. These two boundary conditions, between two media (1) and (2), are automatically taken by the discretization of the equations of Maxwell by choosing:

$$\varepsilon^* = \frac{\varepsilon_1^* + \varepsilon_2^*}{2} \tag{9}$$

$$\mu^* = \frac{\mu_1^* + \mu_2^*}{2} \tag{10}$$

The two absorbent limits in the z direction are given by equations:

$$E_y^{n+1}(i,0) = E_y^n(i,1) + \frac{c\Delta t - \Delta z}{c\Delta t + \Delta z}\left[E_y^{n+1}(i,1) - E_y^n(i,0)\right] \tag{11}$$

$$E_y^{n+1}(i,k_{max}) = E_y^n(i,k_{max}-1) + \frac{c\Delta t - \Delta z}{c\Delta t + \Delta z}\left[E_y^{n+1}(i,k_{max}) - E_y^n(i,k_{max}-1)\right] \tag{12}$$

The difference between the positions of the absorbent limit and that of the sample in the guide of waves must be sufficient so that only the fundamental mode TE_{10} could propagate, the higher-order modes in the waveguide can in this case be neglected. The selected input port is then excited by its TE_{10} modal distribution.

To calculate the S_{ij} parameters at the reference planes of the rectangular waveguide at X-band frequencies, an excitation in the form of a sinusoidal modeled by a Gaussian pulse is used. After an appropriate number of time iterations, a stable distribution is then obtained, and the DFT algorithm can be applied in order to produce the complex amplitude of the desired field as a function of the corresponding frequency.

B. Inverse Problem

To achieve the complex permeability and permittivity of the magnetic material monolayer, we use the "Fminsearch" function implemented on MATLAB [10] which is based on the Nelder-Mead simplex algorithm [11,12]. This function solves the problems of nonlinear multivariate optimization without constraint, which finds the minimum of one scalar function of several variables from an initial estimate of the permeability complex relative and complex relative permittivity such that ε_r^*=1.5-j 0.05 and μ_r^* =1.5-j 0.05.

The error function to be minimized with "Fminsearch" is the sum of the squares of the errors between the measured and calculated S_{ij} parameters which is written as follows:

$$f(\varepsilon_r^*, \mu_r^*) = |S_{11c} - S_{11m}|^2 + |S_{21c} - S_{21m}|^2 \quad (13)$$

III. NUMERICAL RESULTS

A. Direct Problem

1. Magneto-dielectric Material

To validate the 2D-FDTD calculation, a magnetic sample with relative permeability complex μ_r^*= 5-j 0.05 and complex permittivity ε_r^* = 2.3-j0.02, of thickness L = 1.5mm is placed in a standard WR90 waveguide where L_0 = 2.5mm (see Fig. 2). With the values of the 2D-FDTD parameters: for the spatial variables Δx = 1.143mm, with 1 <i <20, and Δz = 0.25mm, with 1 <k <36, so that L_0 and L are multiples integers of Δz, finally for time: Δt = 0.73319 ps and n_{max} = 2500.

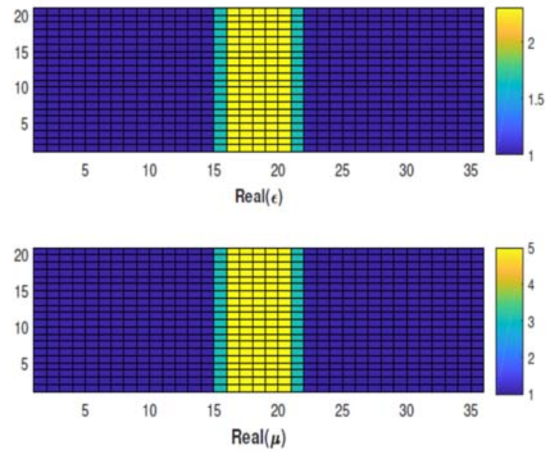

Fig.2. Simulation of a TiC monolayer sample in a waveguide standard WR90.

Fig. 3 represents the component of the electric field in the guide, in the presence of the sample, at the reference planes: k = 5 at the input of the waveguide and k = 31 at exit. This clearly shows the stability and convergence of our 2D-FDTD program according to equations (6) and (8). We also find that the excitation used is well in the form of a sinusoid modulated by a Gaussian.

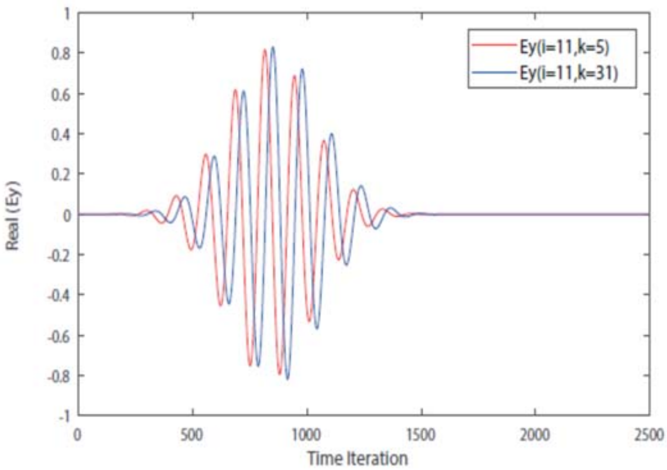

Fig.3. Evolution of the electric field in the standard WR90 waveguide at over time.

Fig. 4 and Fig. 5 represent the real part of the component of the field electrical in all the waveguide in two particular instants n = 817 then n = 852 chosen according to Fig. 3 where the input or output fields are maximum.

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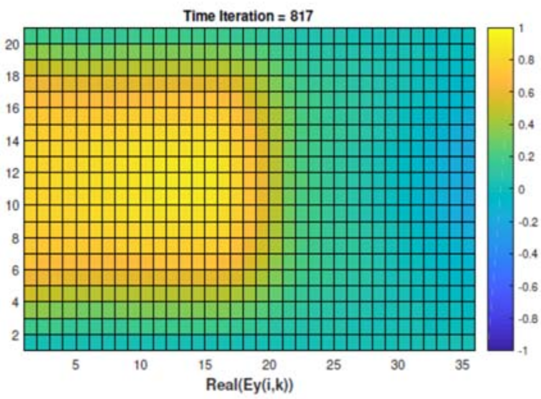

Fig.4. Spatial representation of the electric field in the waveguide at n = 817.

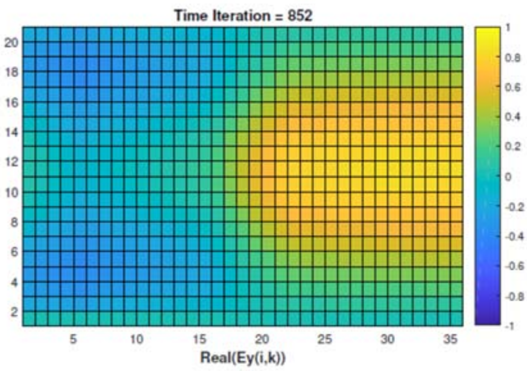

Fig.5. Spatial representation of the electric field in the waveguide at n = 852.

Finally, Fig. 6 shows a good agreement between the parameters S_{ij} deduced by the procedure presented in works [13,14] with those simulated by the HFSS software.

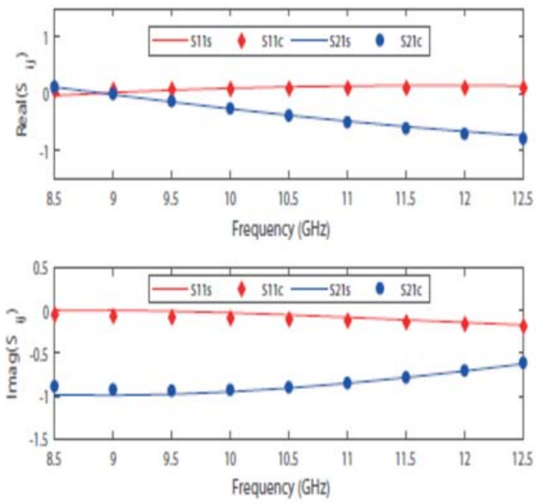

Fig.6. S_{ij} parameters of a single layer sample of TiC in a standard WR90 waveguide.

2. Non-magnetic Material

The 2D-FDTD calculation is also applied to an FR4 Epoxy dielectric sample not magnetic, $\mu_r = 1$, with complex relative permittivity $\varepsilon_r^* = 4.5(1\text{-}j0.02)$, of thickness L = 1.6mm placed in a standard WR90 waveguide where L_0 = 2.4mm and a = 22.86mm. Using the steps $\Delta x = a / 20 = 1.143$mm, $\Delta z = 0.2$mm and the number of time iterations $n_{max} = 2500$.

Fig. 7 shows a good agreement between the parameters S_{ij} calculated with those simulated by the HFSS software.

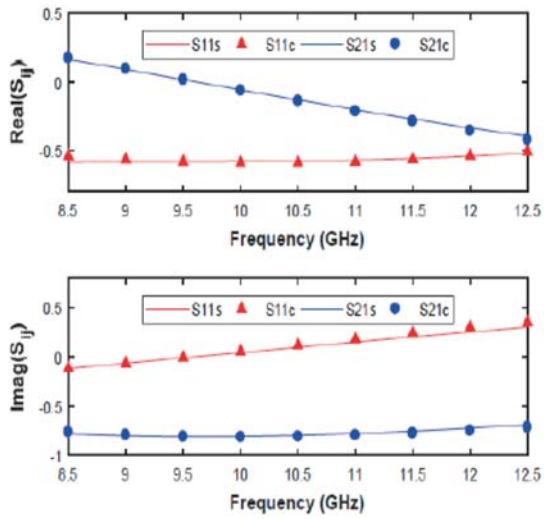

Fig.7. S_{ij} parameters of a monolayer sample of FR4 Epoxy in a standard waveguide WR90.

B. Inverse Problem

To validate the reverse problem, using the procedure described in section (II), the complex permeability and the complex permittivity of a monolayer sample are estimated.

We consider the measurement system in Fig. 8.

Fig.8. The measurement system.

First of all, we applied this method to simultaneously estimate the complex permeability and complex permittivity of a non-dielectric sample magnetic like FR4 Epoxy with a thickness L = 1.6mm. The initial guess of the complex permeability was wall μ_r^*=1.5-j 0.005 and that of the complex permittivity was ε_r^*= 1.5-j0.005.

The results obtained for FR4 Epoxy are shown in Fig. 9 and Fig. 10, it is clear that the mean of the complex permeability obtained wall μ_r^*= 1.004-j0.006 and the mean complex permittivity ε_r^*= 4.427-j0.093 show the dielectric character non-magnetic from FR Epoxy. The results obtained for FR4 Epoxy by the method developed are in good agreement with those obtained in [7].

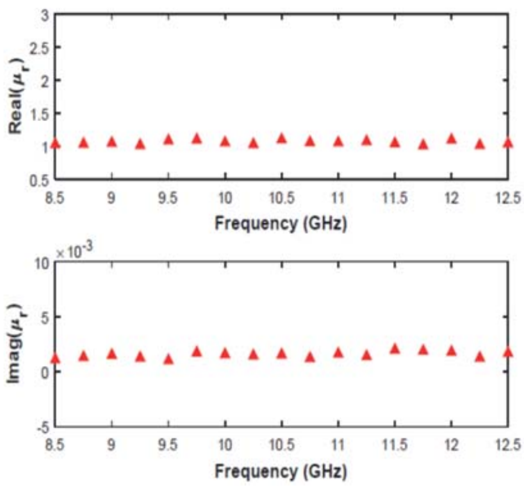

Fig.9. Complex relative permeability of FR4 Epoxy obtained by the method 2D-FDTD combined with the Nelder-Mead algorithm.

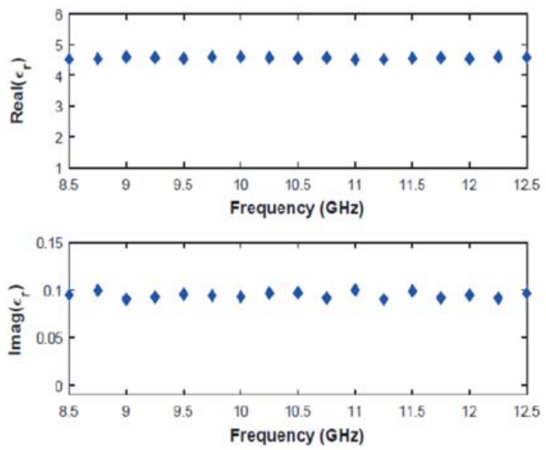

Fig.10. Complex relative permittivity of FR4 Epoxy obtained by the method 2D-FDTD combined with the Nelder-Mead algorithm.

Then, we used this method in order to estimate the complex permeability and the complex permittivity of a magnetic sample of Titanium Carbide (TiC) in powder, with thickness L=1.5mm.

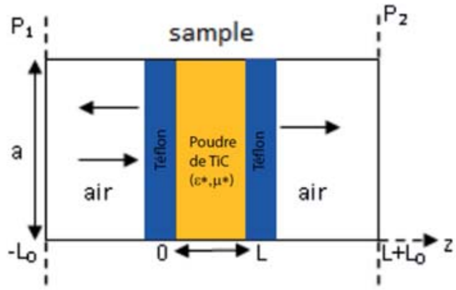

Fig.11. A single layer sample of powdered TiC surrounded by two layers of Teflon in the WR90.

To perform measurements on this type of material, we have placed two layers of Teflon thickness L_1 = 0.5mm on each side of the measuring cell (the holder sample) to make it watertight (Fig. 11). The two Teflon samples are known complex permittivity of ε_r^*=2.08-j0.0021. We used the same assumptions initializations of the complex relative permeability μ_r^*=1.5-j0.005 and of the complex relative permittivity ε_r^*=1.5-j0.005.

The results obtained are plotted on Fig. 12 and Fig. 13, we notice that the TiC sample is a magnetic material of average complex permeability about μ_r^*=5.000-j0.050 and average complex permittivity equal to ε_r^*=2.344-j0.025.

These results are in good agreement with those obtained by another method presented in [6] with a relative error of less than 3%.

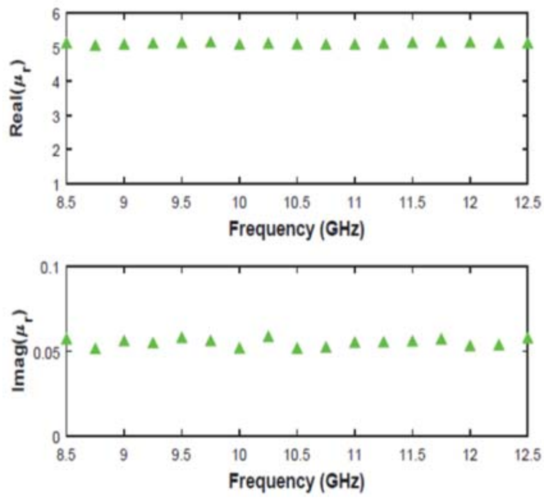

Fig.12. Complex relative permeability of TiC obtained by the 2D-FDTD method combined with the Nelder-Mead algorithm.

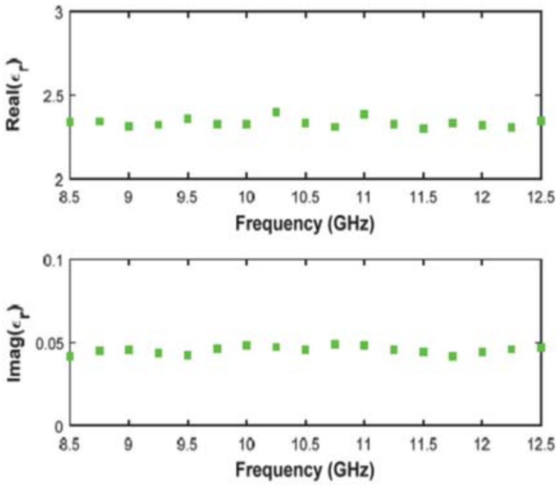

Fig.13. Complex relative permittivity of TiC obtained by the 2D-FDTD method combined with the Nelder-Mead algorithm.

The results presented in Table 1 shows a good agreement between the average values of the measured complex permeability and permittivity with those in [6] and [7]. We remark that the average error is lesser than 3% in real part but it can reach 16% in imaginary part because the air gaps between the sample and the waveguide.

Table 1: Measured values of complex permeability and permittivity with relative errors compared with works in [6] for FR4 Epoxy and [7] for Titanium Carbide.

Material	Complex permeability μ*		% error		Complex permittivity ε*		% error	
	Refs	This work	μ'	μ''	Refs	This work	ε'	ε''
FR4 [6]	1.001-j0.005	1.004-j0.006	<1%	16%	4.450-j0.090	4.427-j0.093	<1%	2%
TiC [7]	5.080-j0.053	5.000-j0.050	<2%	6%	2.400-j0.023	2.344-j0.025	2.3%	10%

IV. CONCLUSION

In this paper, we applied the numerical method 2D-FDTD for calculate the S_{ij} parameters of a single-layer magnetic material, using a guide WR90 standard rectangular wave in X-band. Then the 2D-FDTD numerical method combined with the Nelder-Mead algorithm is used to simultaneously estimate the complex relative permittivity and the permeability relative complex by matching the measured and calculated S_{ij} parameters. The results obtained show that the method developed makes it possible to characterize dielectric and magnetic materials in solid and powder form. Also, this technique can be generalized for multilayers magneto-dielectric sample.

REFERENCES

[1] E. Lin, "A 94-GHz 0.35-W power amplifier module", *IEEE Transactions on Microwave Theory and Techniques*, Vol. 45, No. 12, pp. 2418-2423, December 1997.

[2] G. Avitabile, A. Cidronali, C. Salvador, "Equivalent Circuit Model of GaAs MMIC-Coupled Planar Spiral Inductors", *John Wiley & Sons*, *IJMMWCAE*, Vol. 7, pp. 318-326, 1997.

[3] E. Bènevent, K. Garello, D. Cros, B. Viala, "High permeability and high permittivity heterostructures for the miniaturization of radiofrequency components ", *IJMWT*, Vol. 1, pp. 455-460, December 2009.

[4] K. Ozden, O. M. Yucedag, H. Kocer, "Metamaterial based broadband RF absorber at X-band", *IJEC*, Vol. 70, Issue 8, pp. 1062-1070, May 2016.

[5] Monteiro, Emelia dos Santos, "Nanoparticles of Ni_{1-x} Zn_x Fe_2O_4 used as Microwave Absorbers in the X-band", *Materials Research*, Vol. 22, August 2019.

[6] H. Wang, D. Zhu, W. Zhou, F. Luo, "Electromagnetic and Microwave Absorption Properties of the Carbonyl Iron/TiC Hybrid Powders in the X Band", *IJME*, Vol. 2, No. 1, March 2016.

[7] H. Elmajid, J. Terhzaz, H. Ammor, " Optimization Technique to Estimate the Complex Permittivity of Dielectric Materials at X-Band Using Rectangular Waveguide", *IJAER*, Vol. 9, No. 24, pp. 26709-26718, December 2014.

[8] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media", *IEEE Trans. Antennas Propagation*, Vol. 14, pp. 302-307, May 1966.

[9] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time domain electromagnetic field equations", *IEEE Trans. Electromagn. Compat*., Vol. 23, pp. 377-382, November 1981.

[10] Optimization Toolbox User's Guide, *The MathWorks*, *Version 9.2*, 2017a.

[11] Nelder, J. and R. Mead, "A simplex method for function minimization", *Computer SXSX Journal*, Vol. 7, 1965.

[12] Gao, Fuchang & Han, "Implementing the Nelder-Mead simplex algorithm with adaptive parameters", *Computational Optimization and Applications*, Vol. 51, pp. 259-277, May 2012.

[13] Terhzaz, J, H. Ammor, A. Assir, A. Mamouni, "Application Of The FDTD Method To Determine Complex Permittivity Of Dielectric Materials At Microwave Frequencies Using A Rectangular Waveguide", *Wiley*, Vol. 49, No. 8, pp. 1964-1968, August 2007.

[14] Zhiyuan Yu, "A New Method Of S-Parameter Extraction From The FDTD In Analyses Of Microstrip Circuit Discontinuities*", John Wiley & Sons, Inc, Microwave And Optical Technology Letters*, Vol. 16, No. 3, pp. 162-163, October 1997.